

Pattern Formation in Active Fluids

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Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

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Approval Sheet

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Abstract

Active self organization of structure is ubiquitous in nature. In a seminal work, Turing proposed a general reaction diffusion mechanism for pattern formation [2]. The contractile stress generation and resultant pattern formation in actin-myosin cortical layer lying close to cell membrane, controls from cell motility to tissue development. Considering this cortical layer as an active fluid, we first show steady state pattern formation due to competition between diffusive and active contractile processes, following a recent work done by the Jülicher group [1]. Starting from a uniform distribution, with increasing activity the system shows a non-equilibrium phase transition to density modulated state. We further investigate the impact of active to passive transformation of the fluid in a stress dependent manner.

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Chapter 1

Turing and pattern formation

In 1952, A.M. Turing proposed a reaction- diffusion mechanism to study pattern formation in developing organisms [2]. Pattern formation includes genesis of different organs starting from a spherically symmetric zygote. Several recent experimental studies focussed on zebra fish spinal cord development, or drosophila wing development. Turing proposed that reaction – up or down regulation of a chemical species – and differential diffusivity are the principal cause for the observed patterns. However, many of the biologically relevant processes are mechano-chemical, where system generated mechanical stresses are also expected to affect the pattern formation and hence should be included in the discussion. Such systems, although may be homogeneous at first, may later develop patterns due to self organized instability.

To understand our perspective, it is important to first outline a specific biological example [1].

1.1 Biological Context of the current problem

Let us consider the cortical layer of cytoplasm in an animal cell. It consists of actin filaments, myosin motors, and several cross-linking proteins. Myosin is a motor protein, which might be considered as a self-propelled particle capable to move only on actin filament tracks. Apart from carrying cargo, myosin-II motors can form big bundles that cross-link more than one actin-filament to generate shear stress on actin-filament network. The motor proteins undergo binding unbinding kinematics, rendering a fluidity to the network at long time scales. The transport or stress generation due to motor proteins is an active process, involving stochastic hydrolysis of a chemical fuel called ATP.

Three types of cytoplasmic motor proteins are known. Among these myosins move on actin filaments alone, while dyneins and kinesins use microtubules as tracks. The motor proteins can walk or self propel on this track forward from one binding site to the next carrying their cargo with them, where the forward direction is determined by the polarity of the track, by utilizing the chemical potential from repeated cycles of hydrolysis of ATP to ADP and phosphate

$$\Delta\mu = \mu_{ADP} + \mu_{phosphate} - \mu_{ATP}.$$

Each cycle of binding and release must propel them forward in a single direction along a filament to a new binding site on the filament [3]

These proteins are active in the state when they are attached to their tracks. While unattached to the track they diffuse as a passive element of the fluid and can no longer self propel. In one variant of our model, we incorporate a coarse-grained version of the above-mentioned attachment-detachment process through a stress dependent rate in which a portion of active gel becomes passive.

In the analysis performed in the next chapters we treat the system as one-dimensional. Extending the analysis to higher dimensions is straight forward. Apart from the ease in performing the calculation, the choice is motivated by the fact that a real cortical layer of a two dimensional flat cell turns out to be quasi-one dimensional.

Chapter 2

Hydrodynamic Models

In this chapter we will consider two hydrodynamic models. The first method involves a single active fluid whose concentration is a conserved quantity, or in other words, is always linked to the actin filaments. In the second model, we consider that the actin-myosin fluid can be in two states, one is active and the other is passive. The local concentration of the fluid can change states stochastically. Moreover the active- passive transition rate depends exponentially on the local stress – a higher local stress increasing the passive component.

2.1 Single regulator of active stress

Considering a single active fluid with regulator concentration $c(x,t)$ actively generating hydrodynamic flows in a thin film of active fluid where movements occur in one dimension only. The total concentration or total number of such elements is conserved, hence giving a conservation equation of concentration[1]

$$\partial_t c = -\partial_x j \tag{2.1}$$

with j being given by

$$j = -D\partial_x c + vc$$

where $D > 0$ or

$$\partial_t c = D\partial_x^2 c - \partial_x(vc)$$

The first term in j is the usual diffusive part which is present irrespective of the activity of the fluid. If there is a maxima of concentration, $\partial_x^2 c < 0$ the fluid flows from maxima to the minima and vice versa. The second part of the flux, j is the advective part, present on the account of activity of the fluid. The bulk flow velocity, v is generated due to self propulsion of the active fluid.

The total stress generated consists of viscous part and active part depending on the concentration of the regulator which is positive for contraction[1]

$$\sigma = \eta \partial_x v + \zeta \Delta \mu \quad (2.2)$$

here η is the bulk viscosity, and accounts for stresses generated due to differential velocity along the system. The actively generated stresses is represented by $\zeta \Delta \mu$ where ζ is a proportionality factor and $\Delta \mu$ is the change in potential in one cycle of hydrolysis and is active fluid concentration dependent. This active stress, hence, can also be written as $(\zeta \Delta \mu)_0 f(c)$.

Neglecting inertial forces, valid on a cellular scale, which are small in magnitude[1]

$$\partial_x \sigma = \gamma v \quad (2.3)$$

where γ is the friction coefficient to account for the motion against the membranes.

The total amount of regulator is conserved and average concentration for fluid in length L is given by c_0

$$c_0 = \frac{1}{L} \int_0^L dx c(x, t) \quad (2.4)$$

The homogeneous solution: $c(x)=\text{constant}$, $v=0$ satisfies the above equations and hence is a steady state solution with constant concentration (a particular although trivial example would be $c=0$, $v=0$ which is the absence of any active fluid), as the diffusive and advective components independently goes to 0

$$\partial_t c = 0$$

$$j = 0$$

$$\partial_x \sigma = \gamma v = 0$$

the stress is a constant and is only dependent on the ATP hydrolysis.

2.2 Steady state solution

A steady state solution is represented by a constant in time velocity $v(x, t) \equiv v(x)$ and concentration $c(x, t) \equiv c(x)$ at any point occupied by the fluid. Hence, for a non homogeneous steady state, $\partial_t v = 0$, and $\partial_t c = 0$.

Assuming a constant diffusion coefficient, and using 2.1

$$-\partial_x j = 0$$

$$j = j_0$$

implying

$$v = D \partial_x \ln c + j_0 / c \quad (2.5)$$

for no flux boundary condition $v(0)=0$ and $\partial_x c(0) = 0$ requiring that $j_0 = 0$ because

$$v(0) = D \partial_x \ln c + j_0 / c = 0$$

$$\partial_x c = -j_0 / D = 0$$

Assuming $\gamma = \text{constant}$ in space we can solve as follows

$$\partial_x \sigma = \gamma (D \partial_x \ln c) + \gamma j_0 / c$$

and integrating wrt x ,

$$\sigma = \sigma_0 + \gamma D \ln c + \gamma j_0 \int \frac{dx}{c(x)} \quad (2.6)$$

if we were to integrate from 0 to L we would get

$$j_0 \int_0^L dx / c(x) = \frac{\sigma(L) - \sigma(0)}{\gamma} - D \ln \frac{c(L)}{c(0)} \quad (2.7)$$

since $c(L) = c(0)$ and $\sigma(L) = \sigma(0)$ in periodic boundary conditions we get that j_0 should be zero in both the conditions.

Thus for both the boundary conditions we have from the non homogeneous steady state solution of velocity

$$v = D\partial_x \ln c = D\partial_x u \quad (2.8)$$

$$u \equiv \ln c$$

just substituting this expression in the expression for the total stress involving the unknown term of chemical potential change and using $j_0 = 0$

$$\partial_x^2 u = \frac{\gamma}{\eta} u + (D\eta)^{-1} [\sigma_0 - \zeta \Delta \mu(u)] \quad (2.9)$$

$\ell \equiv \sqrt{\eta/\gamma}$ is a hydrodynamic length.

Equation 2.9 can be seen as the force balance equation with the right hand side playing the role of $-\partial_u V(u)$ since the equation looks like an anharmonic oscillator where the total energy is conserved the Hamiltonian[1] equals the total energy and can be written as

$$H(u, \partial_x u) = 1/2(\partial_x u)^2 + V(u) \quad (2.10)$$

where

$$V(u) = -u^2/2\ell^2 - \frac{1}{D\eta} (\sigma_0 u - \int^u du' \zeta \Delta \mu(u')) \quad (2.11)$$

2.3 Linear stability analysis on homogeneous steady state

We have already seen that a homogeneous steady state is a solution. As discussed in section 1, we now consider the stability of the homogeneous equilibrium by a random disturbance.

To do a linear stability analysis on the homogeneous state $c = c_0$ steady state, we consider a perturbation $\delta c = \delta c_0 e^{\lambda(k)t + i k x}$ where $k = \pm \beta n \pi / L$ and n is an integer. Inserting this in the equation

$$\sigma = \eta \partial_x v + \zeta \Delta \mu \quad (2.12)$$

after differentiating this eqn w.r.t x and equating this with γv we get

$$\eta \partial_x^2 v + \partial_x (\zeta \Delta \mu) = \gamma v$$

and using $\zeta \Delta \mu = (\zeta \Delta \mu)_0 f(c)$ with the form of $f(c)$ not specified, and assuming that $f(c)$ does not change appreciably upon perturbing the system, we get the following differential eqn, where $f(c) \sim f(c_0)$

$$(D^2 - \frac{\gamma}{\eta})v + \frac{(\zeta \Delta \mu)_0}{\eta} \partial_c f(c_0) \iota k \delta c_0 e^{\lambda(k)t + \iota k x} = 0$$

whose particular solution is (neglecting the terms involving second order of $(\delta c)^2$)

$$v(x) = (\delta c \iota k (\zeta \Delta \mu)_0 \partial_c f(c_0)) / (\gamma + \eta k^2) \quad (2.13)$$

using this solution in the equation,

$$\partial_t c = -\partial_x j$$

we get the equation for the eigenvalue in terms of velocity and its derivative as

$$\lambda(k) \delta c = -D k^2 \delta c + (c_0 + \delta c) \partial_x v + \iota k \delta c \quad (2.14)$$

where the only term with linear order in δc is $c_0 \partial_x v$ and $-D k^2$ yielding the solution

$$\lambda(k) = -k^2 D (1 - (Pe c_0 \partial_c f(c_0)) / (1 + k^2 \ell^2)) \quad (2.15)$$

Here the Pe number or the Péclet number is defined as the ration of diffusive to advective time scales. $Pe = (\zeta \Delta \mu)_0 / \sqrt{(\eta \gamma)}$. This can be seen from an order of magnitude calculation. $\tau_d = \ell^2 / D$ and $\tau_a = \ell / v$, are the diffusive and advective time scales respectively. Using 2.3 and the definition of ℓ we see that the fluid velocity $v = (\zeta \Delta \mu)_0 / (\ell \gamma)$. As

$$Pe \equiv \frac{\tau_d}{\tau_a}$$

we get $Pe = (\zeta \Delta \mu)_0 / \sqrt{(\eta \gamma)}$.

A positive $\lambda(k)$ (for some value of k), signifies spontaneous emergence of flow or instability in

the homogeneous steady state flow, i.e. for

$$\frac{Pec_0 \partial_c f(c_0)}{1 + (\beta \pi \ell / L)^2} > 1 \quad (2.16)$$

otherwise indicating a stability of the homogeneous solution or if it is equal to 1, the linear stability analysis fails. This shows how big is the dependence of stability is on Pe or the active flow which is why for Pe=0 or passive flow there are no such observed phenomenon of pattern formation.

The graphical representations of these equations is the subject of next chapter.

2.4 Active-passive transformations

Here we will consider the linking and delinking of the motor proteins from the tracks. Let α be the linking and β delinking rates for the active part (c^*). Consider $\beta = \beta_0 e^{|f_{el}/fd|}$ for some constants β_0 and fd . A zeroeth order approximation to the stress dependence of the β we consider it to be a constant or β_0 .

For the conservation equations of concentrations of active (c^*) and passive (c) parts of the fluid,

$$\partial_t c^* = D \partial_x^2 c^* - \partial_x (v c^*) + \alpha c - \beta c^* \quad (2.17)$$

$$\partial_t c = D \partial_x^2 c - (\alpha c - \beta c^*) \quad (2.18)$$

The passive part of the fluid does not carry any velocity from the active part after it is decoupled i.e as soon as the link is broken the active fluid come to rest. The decoupling proteins should not interfere with the other part of the equation hence the time scales of decoupling must be faster than diffusive time scales $\tau_d = \ell^2/D$ or advective time scales $\tau_a = \ell/v(x, t)$.

For steady state solution:,

Adding the two equations, integrating and solving for velocity gives,

$$v = j(t)/c^* + D \partial_x (\ln c^*) + (D/c^*) \partial_x c \quad (2.19)$$

and the stress by,

$$\sigma = \sigma_0 + \gamma D \ln c^* + D \int \frac{\partial_x c}{c^*} dx \quad (2.20)$$

The forms of both of the above equations is similar to the ones discussed in single regulator model of section 2.1, except for the last term in the two expressions.

2.5 Linear stability analysis on homogeneous steady state for active-passive transformation

Starting with homogeneous flow with the steady concentrations c_0 and c_0^* and perturbing by an amount δc and δc^* which in Fourier space has same spatial components k but different temporal components λ and λ^* .

The equations of stress and gradient of stress remains unchanged as of now hence the expression for velocity remains same with the change that $\text{grad} f(c, c^*)$ will involve two terms,

$$v = \frac{\iota k (\zeta \Delta \mu)_0 (\partial_{c^*} f(c, c^*) \delta c^* + \partial_c f(c, c^*) \delta c)}{\eta k^2 + \gamma} \quad (2.21)$$

since the equations used till now did not involve α or β the velocity is explicitly independent of them.

Substituting the expression for velocity in the equations for concentrations and using only the linear terms, we get two equations

$$\left(\lambda^* + k^2 D + \beta + \frac{k^2 D \gamma P e \partial_{c^*} f(c, c^*) c_0^*}{\eta k^2 + \gamma} \right) \delta c^* = \frac{k^2 D \gamma P e (\partial_c f(c, c^*) c_0^* + \alpha)}{\eta k^2 + \gamma} \delta c + (\alpha c_0 - \beta c_0^*) \quad (2.22)$$

and

$$(\lambda + D k^2 + \alpha) \delta c = \beta \delta c^* - \alpha c_0 + \beta c_0^* \quad (2.23)$$

using the concentration equations at steady state flow we get $\alpha c_0 - \beta c_0^* = 0$ hence removing the last two terms in both the equations. This gives us the following form of equations: $a \delta c^* = b \delta c$ of the two equations. This form obtained by linear stability analysis, can not explain the two different exponents λ and λ^* on either side. Hence for linear analysis $\lambda = \lambda^*$. This is obtained by using this

condition in the following equation,

$$\lambda^* + k^2 D + \beta + \frac{k^2 D \gamma P e \partial_{c^*} f(c, c^*) c_0^*}{\eta k^2 + \gamma} = \frac{k^2 D \gamma P e (\partial_c f(c, c^*) c_0^* + \alpha)}{(\eta k^2 + \gamma)(\lambda + D k^2 + \alpha)} \quad (2.24)$$

to obtain,

$$\begin{aligned} 2\lambda = & -(Dk^2 + \alpha + c) \pm \sqrt{\left\{ Dk^2 + \alpha + k^2 D \left(1 + \frac{\gamma P e \partial_{c^*} f(c, c^*)}{\gamma + \eta k^2} \right) + \beta \right.} \\ & \left. - 4 \left(k^2 D + \beta + \frac{k^2 D \gamma P e \partial_{c^*} f(c, c^*)}{\gamma + \eta k^2} \right) (Dk^2 + \alpha) - \frac{k^2 D \gamma P e (\partial_c f + \alpha)}{\gamma + \eta k^2} \right\}} \end{aligned} \quad (2.25)$$

So linearising the system only makes sense when the two concentrations behave similarly to external perturbation. The possibility of different λ s is not ruled out but is not possible when considering the linear response of the system, unless we perform the second order calculations.

Second order calculations:

The velocity equation holds without dropping any terms, i.e.

$$v = \frac{\iota k (\zeta \Delta \mu)_0 (\partial_{c^*} f(c, c^*) \delta c^* + \partial_c f(c, c^*) \delta c)}{\eta k^2 + \gamma} \quad (2.26)$$

which is quite a general expression obtained without dropping any term or considering any specific form of α or β . For simply an ease of understanding, $f(c, c^*) \equiv f(c^*)$. If we do not drop any term while performing the calculations as in the linear case, and keep the highest order term i.e. the second order term, we get

$$\left\{ \lambda^* + Dk^2 - \frac{c_0^* (\zeta \Delta \mu)_0 k^2 \partial_{c^*} f(c^*)}{\eta k^2 + \gamma} + \beta \right\} \delta c^* = \alpha \delta c + \frac{2(\delta c^*)^2 (\zeta \Delta \mu)_0 k^2 \partial_{c^*} f(c^*)}{\eta k^2 + \gamma} \quad (2.27)$$

writing in readable terms,

$$a e^{\lambda^* t} = b e^{\lambda t} + c e^{2\lambda^* t}$$

even if we assume here that $\lambda \neq \lambda^*$ we will never satisfy the equation,

$$\partial_t c(x, t) = D \partial_x^2 c(x, t) - \alpha c(x, t) + \beta c^*(x, t)$$

which is a linear equation,

$$(\lambda + k^2 D + \alpha) \delta c = \beta \delta c^*$$

or we will always have the form, $a\delta c = b\delta c^*$ and hence

$$\lambda = \lambda^* \tag{2.28}$$

for α, β constants, regardless of the assumption about $f(c) \sim f(c_0)$.

For real λ the linear stability analysis will mean that δc and δc^* remain small throughout the time scale of analysis. So the maximum time we can analyse using LSA will be some τ_L characteristic of the eigenvalues λ s. Linear stability analysis although very helpful provide limited insights. A numerical analysis is also required to visualise the solutions of this chapter.

Chapter 3

Numerical Analysis

As discussed in the last chapter, the insights provided by the numerical analysis are of tremendous value in any calculation. This chapter will explain what are the expected solutions at what parameter values. These parameters are not provided experimentally and need to be considered beforehand. The next section will show the actual numerical methods implemented in a Fortran 90 program and the problems encountered.

3.1 The expectation

For the equation

$$\partial_x^2 u = \ell^{-2} u + (D\eta)^{-1} [\sigma_0 - \zeta \Delta \mu(u)] \quad (3.1)$$

where $u \equiv \ln c$ and $v = D\partial_x u$, for the steady state concentration c and velocity of the fluid v . We expect steady periodic solutions for periodic boundary conditions given by $c(0) = c(L)$ and $v(0) = v(L)$ for certain values of Pe , with $Pe \equiv (\zeta \Delta \mu)_0 / D\gamma$. There are solutions to be expected for no flux and periodic boundary conditions as well. The dependence of the solutions on the parameters can be explained by considering the Hamiltonian of the system,

$$H(u, \partial_x u) = 1/2 (\partial_x u)^2 + V(u) \quad (3.2)$$

where,

$$V(u) = -u^2/2\ell^2 - \frac{1}{D\eta} (\sigma_0 u - \int_0^u du' \zeta \Delta \mu(u')) \quad (3.3)$$

If $\partial_u^2 V < 0$ for all u where $\partial_u^2 V = \ell^{-2} (Pe \partial_u f - 1)$ then the potential is ever falling and the only

steady state possible is a homogeneous steady state. Péclet number or Pe is a measure of the strength of the active stress to support flow into the peaks. The potential has a valley for only positive values of σ_0 and bigger $Pe(> 10)$ supporting periodic solutions with a number of peaks and valleys. The basic information of whether or not a Pe parameter value can result in non homogeneous solutions is obtained if we plot a graph of $\ell^2 \partial_u^2 V = Pe/(1+u)^2 - 1$.

3.2 Methods of numeric calculations

For steady state solution the equation reduces to a second order ordinary differential equation. The solution of the equation have two main constraints. Firstly the solutions must be periodic and secondly the solution must satisfy the conservation of total amount of regulator, i.e.

$$c_0 = 1/L \int_0^L dx c(x, t) \quad (3.4)$$

This condition is important but not restrictive as any solution which does not satisfy the conservation equation can be scaled to give the integral a predetermined value of c_0 . This can be done since $c(x)$ and $v(x)$ are bounded and continuous functions of x . This condition however should pose significant problems in solving for non steady states, as if we scale at each iteration in the finite element method then it could produce discontinuities in the overall solution.

The periodic condition is a two point boundary value problem, which can be easily handled by shooting methods. The system is two dimensional to solve and hence does not pose any serious computational problem. Runge-Kutta (fourth order) have been used to solve the differential equation in between the boundary points once the initial guess is made. This guess is improved by solving the periodic boundary conditions for its zeroes, $f(u(0)) = u(0) - u(L)$ and $g(u(0)) = \partial_x u(0) - \partial_x u(L)$ utilising the fact that steady state velocity $v = D\partial_x u$, using Newton's method. The integrations performed are calculated by equal spacing Simpson's 1/3 rule. As a reference of the good convergence and stability of the method, the error is shown for a typical case in figure 1, i.e. $c(L) - c(0)$ and $v(L) - v(0)$ on the y axis and the number of iterations on the x-axis

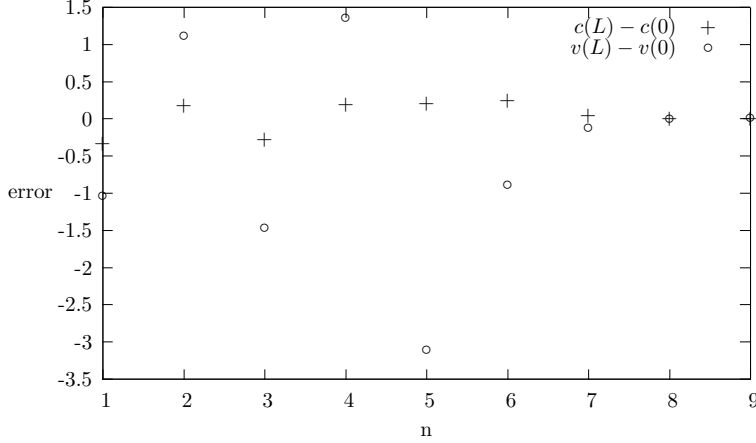


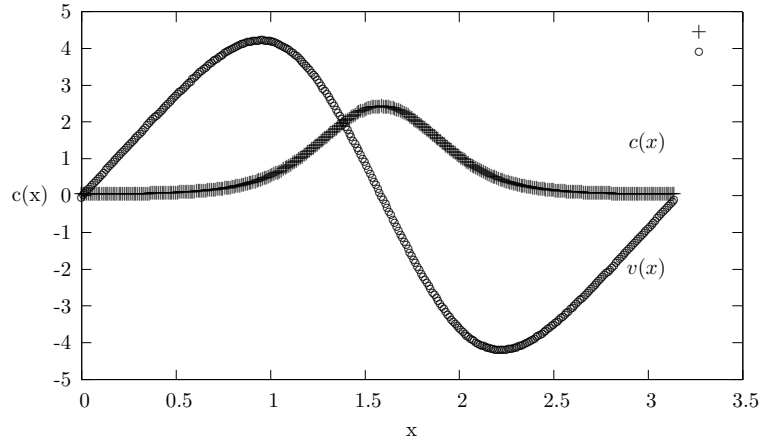
Figure 3.1: $c(L) - c(0)$ and $v(L) - v(0)$ on the y axis and the number of iterations on the x-axis, going to zero for a typical calculation, most others converge taking 10-15 iterations on average

As for the cases of no convergence, we could alter the initial conditions as the method is not globally convergent, i.e. is dependent on the initial conditions the user inputs. There is also a very innate problem encountered: For a system with total length $L/\ell = \pi$ we chiefly encounter a single peak, and for $L/\ell = 2\pi$ we encounter double the number, never vice versa. While both of these should be solutions[1]. Although these solutions are mere extensions of each other and present no new insight to the problem, they are important when we analyse the time dynamics of these two peaked systems which being unstable relaxes to single peaks[1].

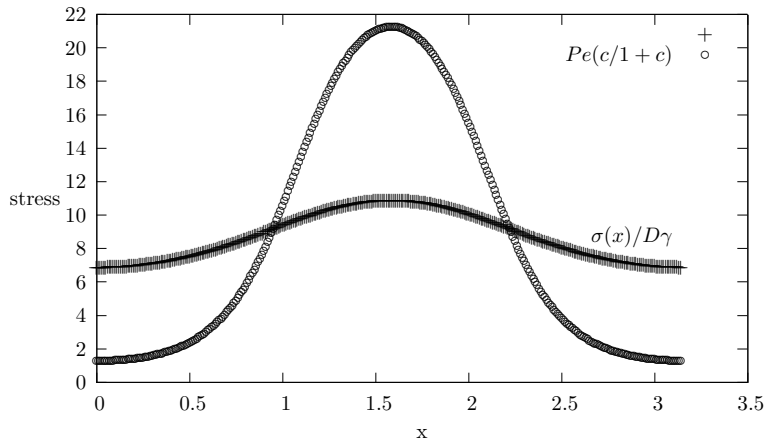
3.3 Graphical representation of solutions for single regulator model

Below are the graphs for different parameter values with no scaling for steady state patterns in single regulator active fluids. In all of these graphs the active stress is in equilibrium to diffusive flow and resistive flow which counteracts the flow of active fluid into regions of high concentrations.

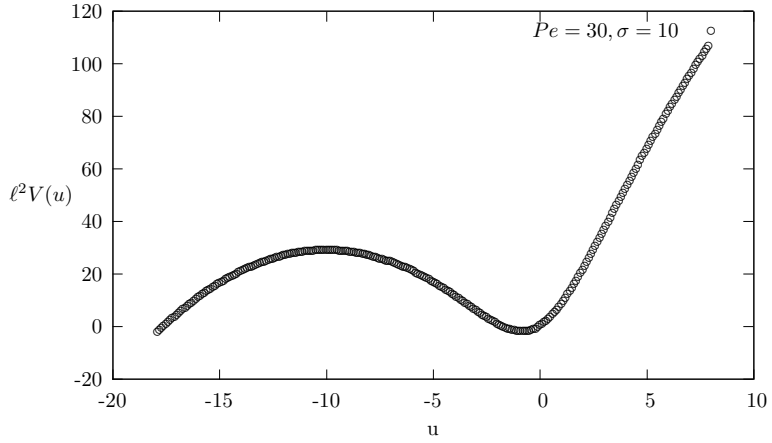
At the end are the graphs for linear stability analysis calculations of equations 2.15 and 2.16



(a)

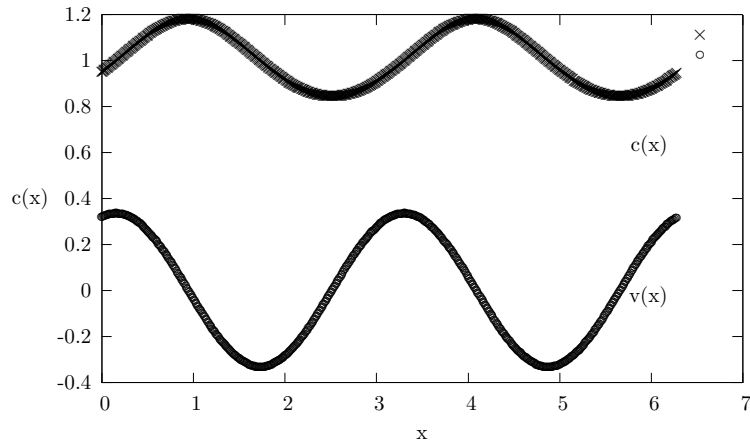


(b)

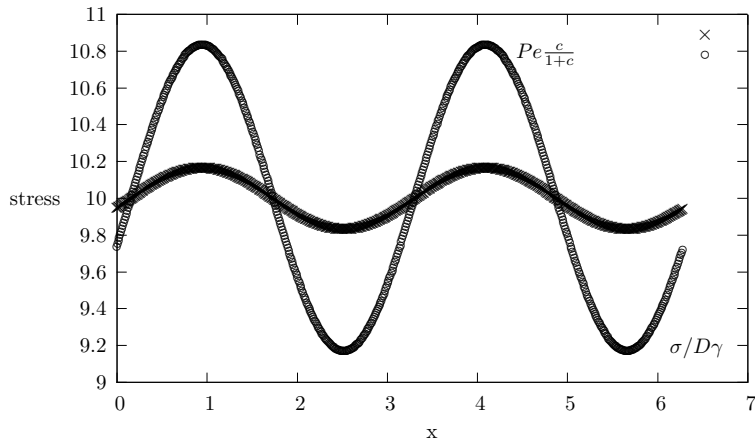


(c)

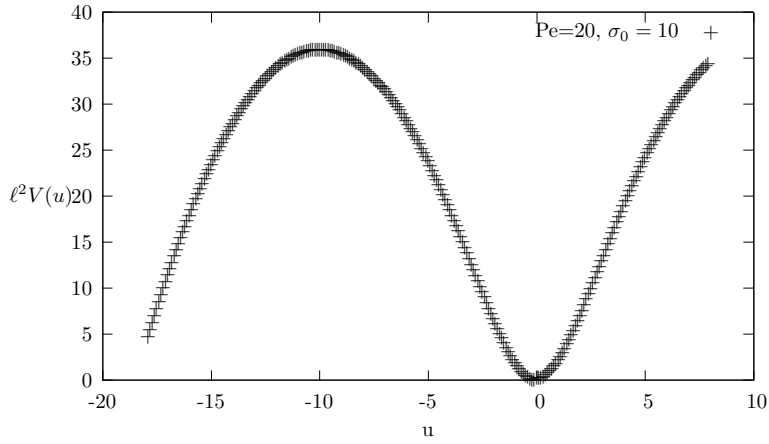
Figure 3.2: Non homogeneous steady states for $Pe=30, \sigma_0 = 10, L/\ell = \pi$ (a) $c(x)$ and $v(x)$ are plotted. $c(x)$ has one maxima and advective flow flows into the peaks, i.e. the velocity flow is towards the peaks (b) the total stress and the active stress is also maximum at the maximum of concentration (c) the potential shows a region where for certain values of Hamiltonian there is a possibility of oscillations



(a)



(b)



(c)

Figure 3.3: For $Pe=20$ and $\sigma = 10, L/\ell = 2\pi$ we observe a two maxima. (a) $c(x)$ and $v(x)$ shows two peaks both with similar features as in 1 peak system, repeated over. (b) The stress also shows similar characteristics (c) The main change is the how much more steep is the slope at the bottom of the potential in close analogy with simple harmonic oscillators

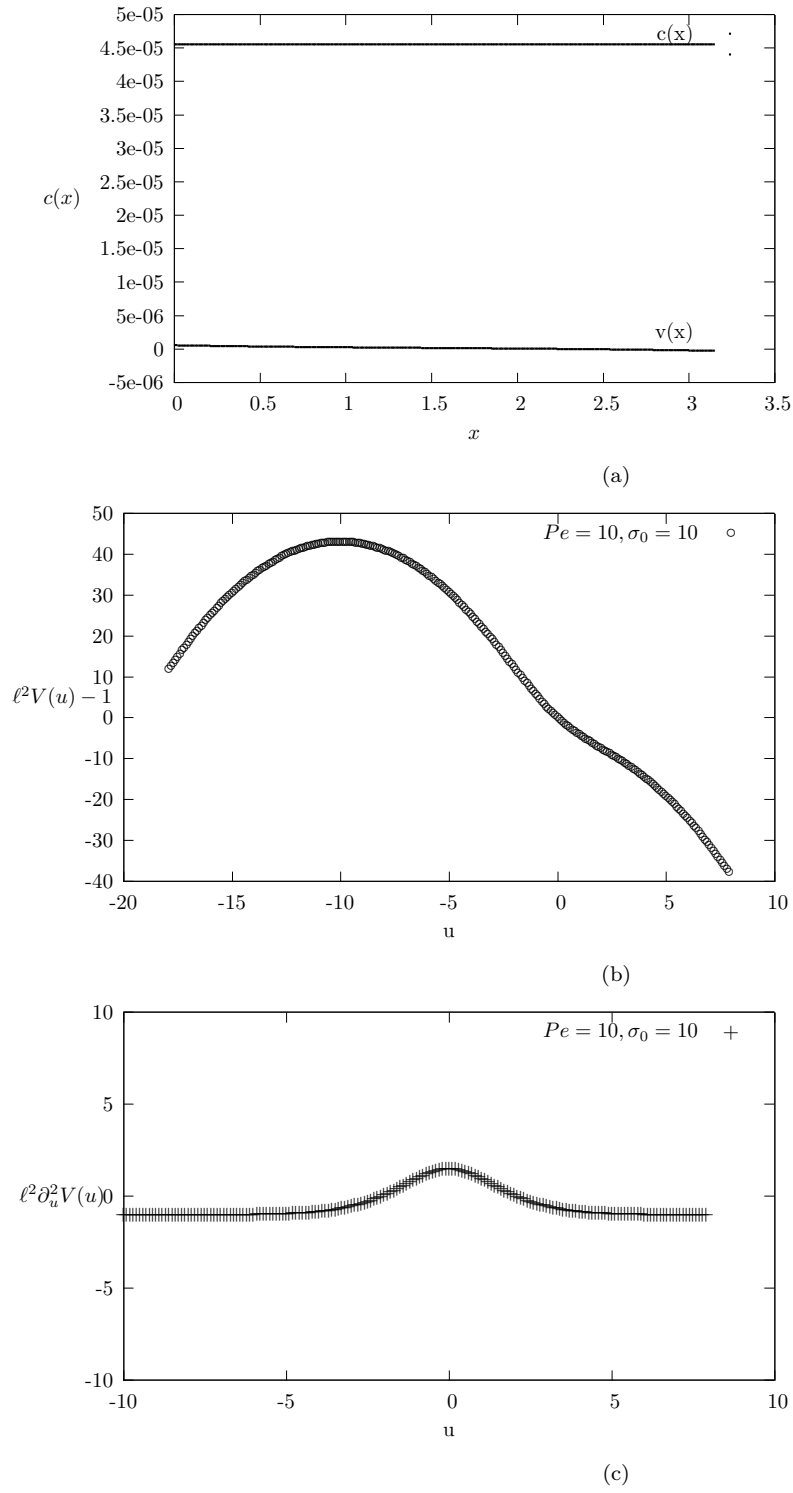
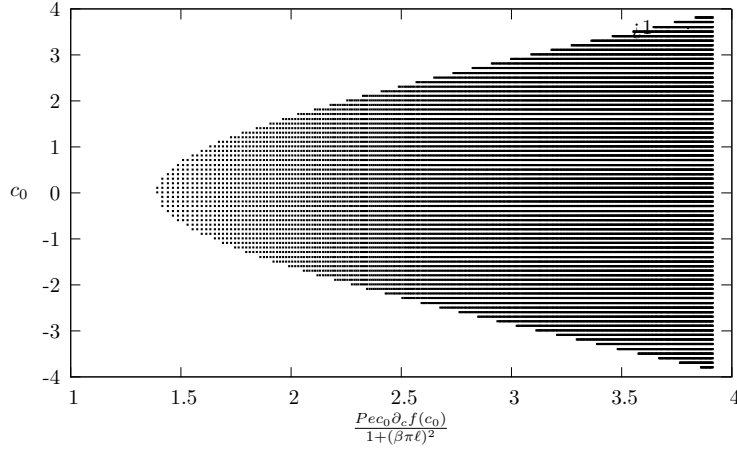
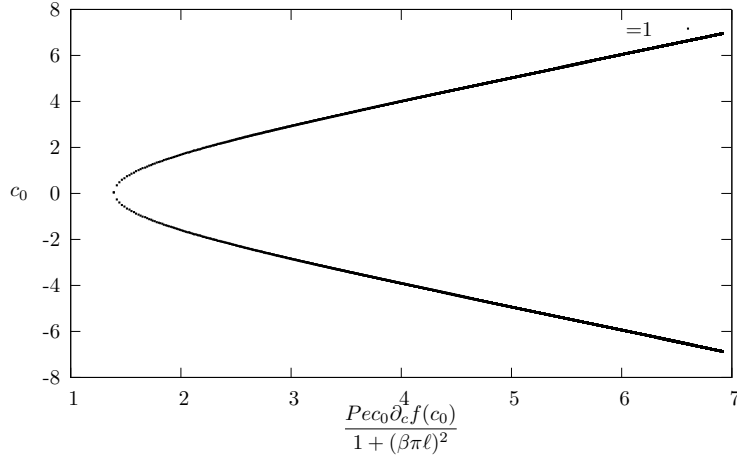


Figure 3.4: Almost homogeneous steady states for $Pe=10$, $\sigma_0 = 10, L/\ell = \pi$ (a) $c(x)$ and $v(x)$ are plotted which are almost homogeneous solutions with no scaling (b) and (c) the potential as a function of u remains concave for most part of the range, barring $u \in [-2.74 : 2.55]$ where we can observe that $\ell^2 \partial_u^2 V(u) - 1 > 0$



(a) The region is blackened where the homogeneous steady states are unstable and $\frac{Pec_0 \partial_c f(c_0)}{1 + (\beta \pi \ell)^2} > 1$ there will be a spontaneous emergence of flow.



(b) For $\lambda = 1$, where the Linear stability analysis fails and we have to consider second order calculations.

Figure 3.5: Linear stability analysis calculations of equations 2.15 and 2.16

Chapter 4

Conclusions

Although we have derived motivation from specific biological examples of motor proteins or cell cortex the methods or the defining equations in both the models are quite general and expect a lot more applications.

It was seen that in the single regulator of active stress model we encountered pattern formation when the active stresses generated were enough to counteract diffusive and frictional forces.

In the linear stability analysis Péclet number played an important role in determining whether a homogeneous steady state is stable or will give rise to spontaneous flows. It also played the role of the defining parameter in numerical calculations. It is not a surprise then that Pe is closely linked to the active stresses generated which are responsible for flows into regions of high concentration resulting in formation of steady patterns. The steady state does not carry any current, and as a result the steady state patterns are time independent.

For the model with transformations of active-passive parts of the fluid, the linear stability analysis calculations turns out to be too tedious to be investigated further. We need to extend the numerical calculation for this active-passive two-state system. We expect the steady state to carry current, and the patterns to be dynamic. It will be interesting to study these patterns in future.

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